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A new tool for experimental investigation of chaos: Theory and applications

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Abstract. The paper is devoted to the description of a new method for the experimental research of chaotic processes based on the modern methods of time-frequency analysis with high resolution. The method does not demand power estimations of a spectrum. The method is based on the analysis of form similarity between a signal and clear sinusoid with certain frequency (form-analysis) that leads to the high method's resistant to noise and to trends. In the experiment the substantive theses of the nonlinear theory of determinate chaos have been confirmed. The new effects, which have been not described earlier, have been shown in the theory. Prospects of use of the form-analysis in various spheres of knowledge have been shown.

Аннотация. Работа посвящена описанию нового метода для экспериментального исследования хаотических процессов, основанного на современных методах время-частотного анализа высокого разрешения. Метод не требует энергетических оценок спектра, а основан лишь на анализе сходства формы сигнала с синусоидой определённой частоты (анализ формы), что приводит к высокой устойчивости его к шуму и трендам. Экспериментально подтвержден ряд положений теории детерминированного хаоса. Получен ряд новых эффектов, не описанных ранее в теории. Показаны перспективы использования анализа формы в различных отраслях знания.

1. Introduction

The concept of structured chaos contrary to the concept of randomness was completely formed at the close of the twentieth century (*Gleick*, 1987). Unfortunately the tools for the chaos' experimental investigations contain not sufficient set of the effective methods. In most experimental cases only the fractal dimensions and the Ljapunov exponents use to decision of the problems. But the original signal must be mapped in a phase space for the method's using. Correct mapping of the experimental signals in phase space is not a trivial task. An embedding procedure must be used for that (*Takens*, 1981). The requisite value of time delay and the correct value of the embedding dimension must be estimated for successful using of imbedding procedure. Despite methods' complexity, a sensitivity of calculations' results to the changes (even to significant changes) of the process' properties is smallish. Therefore the examples of the methods' successful applications in the applied developments were not reached. We have offered a new tool for experimental investigation of chaos, which has high sensitivity to fine time-frequency structure of chaos without preprocessing of an original signal.

2. Theory

For investigation of the structured or determinate chaos we offer a special method of time-frequency analysis based on the wavelet-like time-frequency analysis with high resolution in the time domain and in the frequency domain too. Contrary to existing methods we have estimated not a power of spectrum, but a likeness of a signal's form to the form of clear harmonics with any frequency of the analyzed set. For solving the problem we have constructed the surrogate signals for any frequency at any time from the results of time-frequency analysis. The surrogate signals lose phase information, but hold magnitude information about the frequency components in the time-frequency domain. Finally we have calculated a non-dimensional measure of the likeness between orbits in phase space for surrogate signal and the orbits (circles) for clear harmonic. The non-dimensional measure, which is depended on orbits' area, we have named as a form-index. We can calculate the form-index for all allowed frequencies at any samples of the signal then the manifold of form-indexes is a two-dimensional matrix $\boldsymbol{I}_f(m,n)$, where m – the natural number corresponded to frequency measured in Nyquist frequency (scale); n – the natural number corresponded to the number of the sample at the digital representation of the signal (obviously, that $n_{max} = 2m_{max}$). The values of the array cells belong to the real interval [-1,3]. The two-dimensional matrix $\boldsymbol{I}_f(m,n)$ we will name hereinafter as a form-spectrum of signal and a procedure of matrix calculation as form-analysis.

Let's describe the method step-by-step:

Step1. Carrying out of the signal's high resolution time-frequency analysis. For example, any type of the wavelet-like decomposes (*Demanet, Ying*, 2007) can be used. The spectrum matrix $\boldsymbol{S}(m,n)$ will be received as a result.
Step 2. Constructing of the surrogate signal (*Schreiber, Schmitz*, 1999):

$$p_{ik}(t) = S(i,k)\cos(\omega t) + S(2i,k)\cos(2\omega t) + S(3i,k)\cos(3\omega t) + S(4i,k)\cos(4\omega t).$$

Step 3. Mapping (*Prygunov*, 2003) of the surrogate signal in phase domain (y,x) via embedding procedure:

$$x = p_{ik}(t);\ y = p_{ik}(t - \pi/2\omega).$$

Step 4. Calculating orbits' area A_{ik} and array cell of the form-index matrix $I_f(i,k) = A_{ik}/\pi$.
Step 5. Iteration of steps 2-4 for other i,k in specified intervals.

Below we shall give short comments to choosing of the decomposition.

Wavelet analysis can be considered as the most appropriated way for high-resolution time-frequency analysis of signals. Wavelet decomposition should be understood as an extreme generalization of processes' mapping in the functional space. Therefore all known variants of signals' decomposition, including decomposition via Fourier series, can be considered as a special case of the wavelet decomposition. A wide experience of practical using of wavelet transform in different variants demands a classification and better understanding of wavelet-like decomposition's methods.

First in (*Demanet, Ying*, 2007) the main wavelet-like methods were presented as points on a plane (α,β), where coordinates of the points reflect a major property of the decompositions (Fig. 1): multiscaling in frequency domain ($\sim 2^{-\beta j}$) and in time (spatial) domain ($\sim 2^{-\alpha j}$), respectively. For example, ordinary wavelets have multiscaling in both domains ($\alpha = 1$, $\beta = 1$) whereas Gabor's atoms have not multiscaling ($\alpha = 0$, $\beta = 0$). Therefore, Gabor decomposition do not approach to the digital data reduction, but Gabor has better time-frequency resolution. The wave atoms occupy a middle position. The positions inside the gray triangle are unsuitable for successful decompositions.

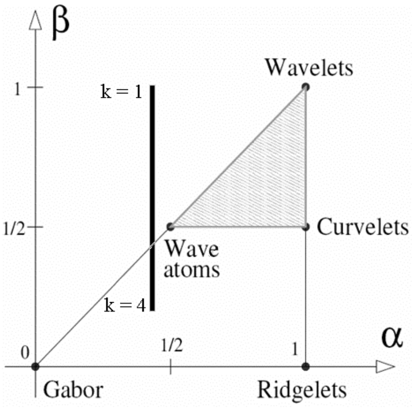

Fig. 1

We have used a special wave basis for signals' decomposition, which can be considered as Gabor wave packets under frequency-depended windows:

$$g_{j,b} = \pi^{-1/4}\, 2^{j/2B} \exp\{-2^{2j/B} h^2 (n-b)^2/2\},$$

where $j = \mathrm{int}[B\log_2(f / f_0)]$, $f_0 = k / 2\sqrt{\ln 2}$ Hz, $B = 3$, $h = 1/S$, S – samples rate in samples per second. By changing the parameter k we can change an effective width of the windows. The offered decomposition can be presented in Fig. 1 as a black band depending on the k value. For reaching of appropriate resolution we have accepted $k = 3$.

Additionally we shall give some comments about the term "surrogate signal".

The method of surrogate signals' compilation was offered in (*Schreiber, Schmitz*, 1999) for statistical recognizing of structured chaos from random processes by a comparison between probability density for an original signal and probability density for a surrogate signal. The surrogate signal has a random phase. Our signal (step 2) is close to the surrogate signal, because our signal "forgot" phase information too. Embedding of the surrogate signal in the phase space (step 3) is a clear procedure, because parameters of embedding for the surrogate signal are trivial.

In Fig. 2 the form-spectrums for two random signals, which have different ways of formation, are presented. The left signal was formatted as pseudorandom series (virtual data) via a formal algorithm; the right signal was formatted as truly random series (physical data) per a sound cart of the personal computer. In the picture signals (top), form-spectrums (middle) and Fourier spectrums of signals (bottom) are presented. The signals contained 320 samples each. The form-spectrum (matrix $\boldsymbol{I}_f(40,320)$) are presented in shading interpretation (MATLAB). Along the ordinate the periods of oscillations in Nyquist period P_N were placed. The dark layers and spots in the picture correspond with quasi-harmonics in the original signal. For better understanding of the pictures four horizontal lines ($8P_N$, $13P_N$, $21P_N$, $34P_N$) were drew in left form-spectrum. The lines' periods form the Fibonacci series and dark layers correspond with them that confirm the method's high effectiveness for the chaos structures' investigation. In accordance with the theory there are two main ways to the chaotic dynamics: Feigenbaum scenario via doubling of periods (*Feigenbaum*, 1978) and Fibonacci scenario via mapping of a circle onto itself in the domain of a critical point GM (golden mean) (*Shenker*, 1982). The method allows recognizing both structures in experimental series without preprocessing. By our investigation for virtual data the Fibonacci frequency structures are typical whereas for physical data mainly the Feigenbaum structures are presented. As an example, right form-spectrum contains double periods, which slowly changed in a time domain.

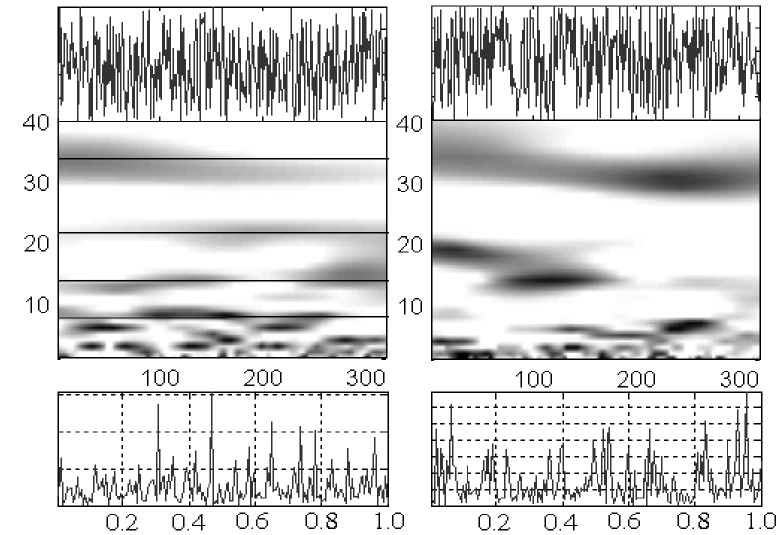

Fig. 2

Fig. 2 presents the distinctions between Fourier spectrums and form-spectrums. As a result of frequency multiscaling, most informative part of the form-spectrum ($5\text{-}40P_N$) corresponds to the narrow low-frequency interval in Fourier spectrums (0.025-0.2). High-frequency part of Fourier spectrum gives good estimation of process' power but not sensitive to variation of the wave form, therefore the methods supplement each other.

3. Applications

3.1. Long-term processes

As an example of long-term processes the meteorological variations were examined. In Fig. 3 the results of form-analysis of ice age temperature change were presented. The original signal in Fig. 3a was the temperature reconstructed by Vostok Ice Core Data for 420,000 Years (*Petit et al.*, 1999). This temperature is calculated using a deuterium/temperature gradient of 9‰/°C after accounting for the isotopic change of sea water. In the picture the horizontal axis is an ice age in kilo years (kYr), the vertical axis for the original signal is the paleotemperature variation (K), the vertical axis for the form-spectrum is the period of oscillations in Nyquist period P_N (here P_N = 2 kYr).

The form-spectrum discloses the long-term quasi-periodic fluctuations of the paleoclimate and the tendency of the frequencies' changes in time. Initial frequency structure (400 kYr ago) was near to Feigenbaum period structure: $10P_N$, $20P_N$, $40P_N$. But after ice age 200 kYr the Fibonacci period structure: $13P_N$, $21P_N$, $34P_N$, $55P_N$ was formed.

The original signal in Fig. 3b was the winter temperature reconstructed by Central Greenland Ice Core Data for the latest 420 Years without data smoothing (*Jones, Mann*, 2004). The picture confirms a noise-immunity of the method. Even for non-smoothed signal the form-spectrum discloses the intricate frequency structure of the original signal. The signal contains only modes with relatively short periods under $40P_N$ (here P_N = 2 Yr). The initial structure (400 Yr ago) is Fibonacci-like structure: $8P_N$, $13P_N$, $21P_N$, $34P_N$. 250 Yr ago the structure became the single-mode ($10P_N$) quasi-periodic structure (see the original signal at this time). After ice age 200 Yr the structure developed into the Feigenbaum-like structure: $12P_N$, $24P_N$.

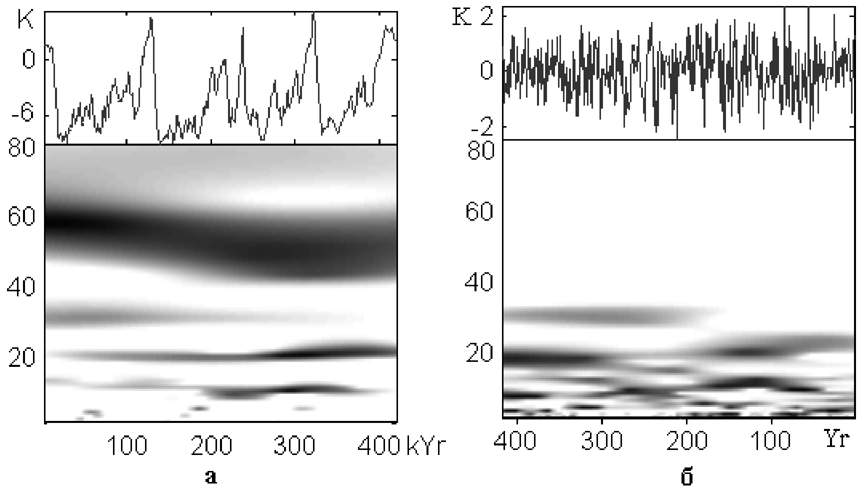

Fig. 3

The results have great importance for a general chaos theory, since a transformation between the base theoretical types of the chaos structure as result of a real processes' evolution has been achieved for the first time.

3.2. Processes in economics

The later investigations in econophysics make it clear that the processes in economics must be considered as structured chaotic processes (*Yakovenko*, 2003). The form-analysis confirms that conclusion.

In Fig. 4 the results of form-analysis for economic processes are presented. For example, we have chosen daily time series (closed values) for two instruments: Standard & Poor 500 Index (S&P500) and currency rate USD/EUR from February 2007 till February 2008.

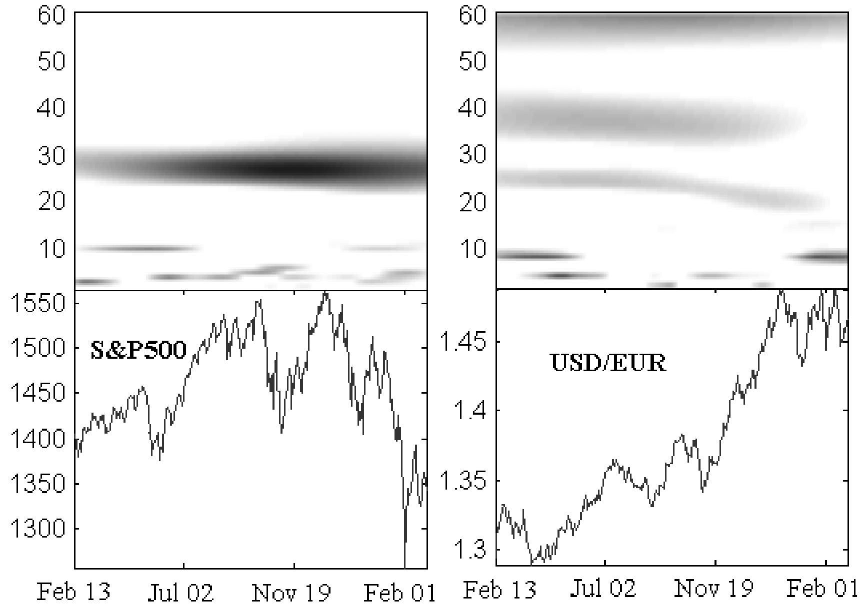

Fig. 4

In the dynamics of S&P there are a lot of short-time nonstationary oscillations with periods less than $10P_N$ (here P_N = 2 day) or with intra-month periods. But the process has the quasi-harmonic oscillation with the period $28P_N$ (about three months). Euro/dollar dynamics shows the Fibonacci-like behavior: periods $25P_N$, $40P_N$, $64P_N$. The last conclusion has great importance for a technical analysis of the markets because the Fibonacci analysis is a traditional tool for markets' forecasting. The form-analysis can be used as an indicator for the traditional Fibonacci analysis' successful applications. The base frequencies of Fibonacci series can be determined via the form-analysis too.

3.3. Medium-term processes in some technologies

Now we examine the applications of the form-analysis to medium-term technological processes. In Fig. 5a the temporal changes of a steam rate (in m^3 per minute) in a powerful heating pipeline are presented (here P_N = 1 minute). The process of heating was in a condition of manual adjusting. In an early stage the process seemed as a Fibonacci-like chaotic process (the horizontal lines mark the Fibonacci-like periods: $11P_N$, $16P_N$, $24P_N$, $37P_N$, the intervals between which correspond with the Fibonacci numbers: $3P_N$, $5P_N$, $8P_N$, $13P_N$). But after 200 minutes the process had evolved to periods doubling's structure ($8P_N$, $16P_N$). The relatively stable stages of the process were remarked after 200 minutes and so the Fibonacci-like chaos can be considered as a deeper chaos than the Feigenbaum-like chaos.

In Fig. 5b the temporal changes of the drilling fluid's pressure (in the technical atmosphere) in the domain of a well bottom are presented (here P_N = 12 minutes). The investigation of nature of non-regular pulsing of the pressure has intense interest for drilling people, because formation fluid's rush is a dangerous incident. The periods' structure of the process contains four main periods: $8P_N$, $11P_N$, $16P_N$, $24P_N$. The intervals between periods ($3P_N$, $5P_N$, $8P_N$) correspond with the Fibonacci numbers. During additional investigation we have made sure that the period $24P_N$ was presented in the signal even without salient impulses, but the periods $11P_N$, $16P_N$ have especially intensified close to the impulses. Therefore evolving of periods $11P_N$, $16P_N$ can be used for pressure impulses' forecasting.

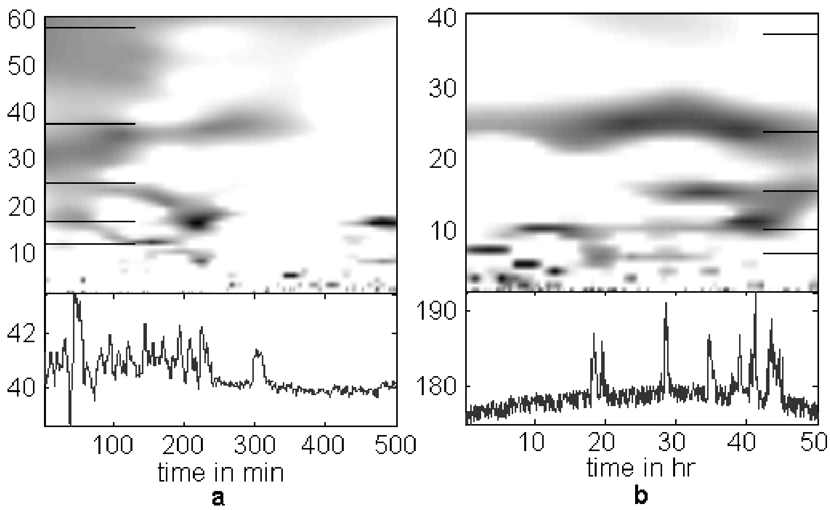

Fig. 5

4. Conclusions

The new method for experimental investigation of chaos has been presented. The method is based:

– on the wavelet-like time-frequency analysis with high resolution;
– on construction of a signal as a sort of the surrogate signals;
– on signal's phase mapping via embedding procedure;
– on the phase traces' estimation by means of the non-dimensional form-index.

The method does not need signal's pre-handling but is resistant to trends and to noise.

The method is applied to investigation of different natural, economic and technical complicated processes.

The similar frequency structure for different time lags, as a fractal effect in structured chaos, has been shown. There are two types of frequency structure: Feigenbaum-like structures formed via periods' doubling; Fibonacci-like structures formed close to the critical point GM and contained the periods which are distributed as a Fibonacci sequence. The Fibonacci-like chaos seems to be deeper and more stable chaos than the Feigenbaum-like chaos. For the first time crossing between the structure types is presented as an evolutionary process.

The method has good perspective to wide using for the investigations of the non-stable dynamic processes or chaotic processes in different spheres of knowledge.

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